Modeling the Human Heart Beat

The period:

$$p = 2L = 1000 \text{ mS}$$

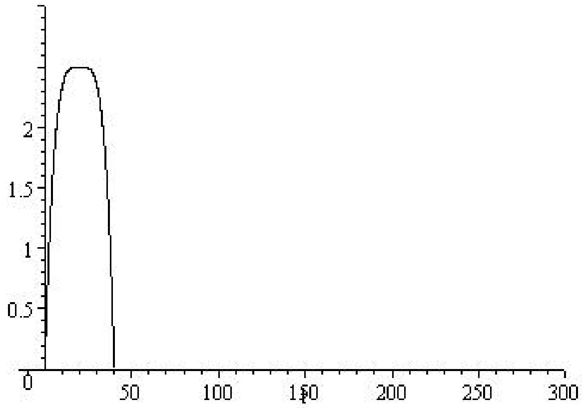
So

$$L = 500$$

Our function is:

$$f(t) = -0.0000156(t-20)^4 + 2.5$$

The graph of the function:



Mean value:

$$\begin{aligned} a_0 &= \frac{1}{L}\int_{-L}^{L} f(t)\,dt \\ &= \frac{1}{500}\int_{-500}^{500} f(t)\,dt \\ &= \frac{1}{500}\int_{0}^{40} (-0.0000156(t-20)^4 + 2.5)\,dt \\ &= 0.16 \end{aligned}$$

Coefficient an:

$$\begin{aligned} a_n &= \frac{1}{L}\int_{-L}^{L} f(t)\cos\frac{n\pi t}{L}\,dt \\ &= \frac{1}{500}\int_{0}^{40} (-0.0000156(t-20)^4 + 2.5)\cos\frac{n\pi t}{500}\,dt \\ &= -4.0\times 10^{-10}(5.57\times 10^{8} n^4 \sin 0.251 n + 8.11\times 10^{10} n^3 \cos 0.251 n \\ &\quad - 1.94\times 10^{12} n^2 \sin 0.251 n + 2.45\times 10^{14} \sin 0.251 n - 3.08\times 10^{13} n \cos 0.251 n \\ &\quad + 8.11\times 10^{10} n^3 - 3.08\times 10^{13} n)/n^5 \end{aligned}$$

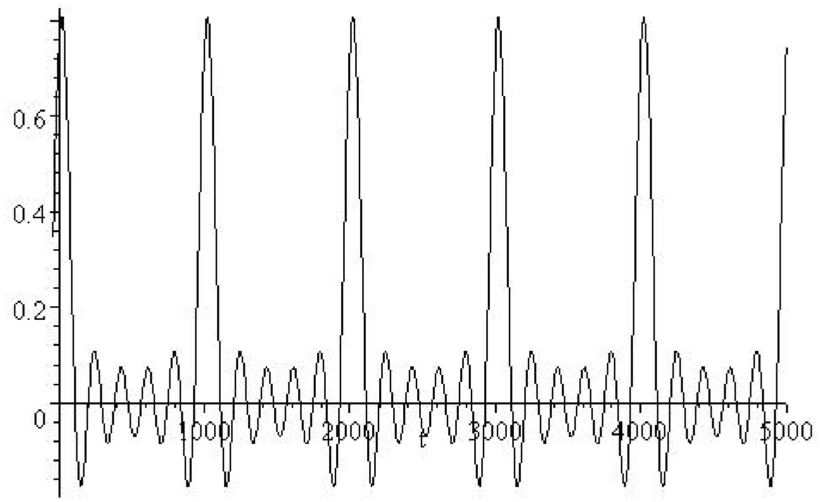
Coefficient bn:

$$b_n = \frac{1}{L}\int_{-L}^{L} f(t)\sin\frac{n\pi t}{L}\,dt$$
$$= \frac{1}{500}\int_{0}^{40}(-0.0000156(t-20)^4 + 2.5)\sin\frac{n\pi t}{500}\,dt$$
$$= 4.0\times10^{-10}(5.57\times10^{8}n^4\cos 0.251n - 8.11\times10^{10}n^3\sin 0.251n$$
$$-1.94\times10^{12}n^2\cos 0.251n + 2.45\times10^{14}\cos 0.251n$$
$$+3.08\times10^{13}n\sin 0.251n - 2.45\times10^{14} - 5.57\times10^{8}n^4 + 1.94\times10^{12}n^2)/n^5$$

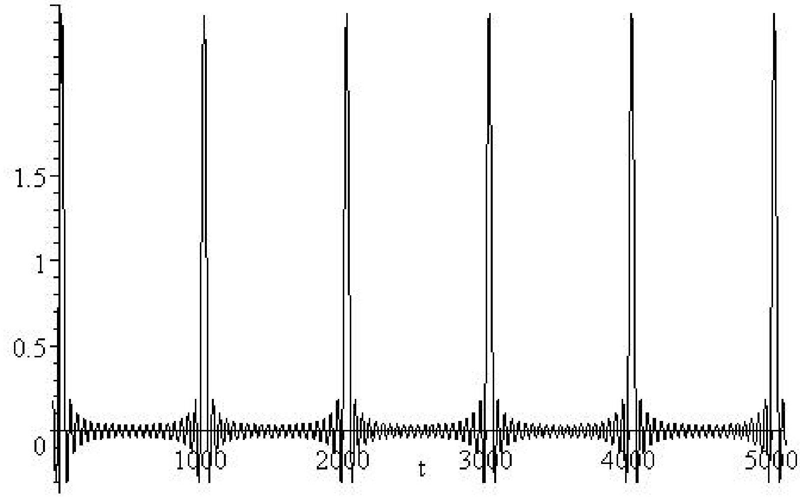
Fourier Series:

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n\cos\frac{n\pi t}{L} + \sum_{n=1}^{\infty} b_n\sin\frac{n\pi t}{L}$$
$$= \frac{0.16}{2} + \sum_{n=1}^{\infty}(-4.0\times10^{-10}(5.57\times10^{8}n^4\sin 0.251n + 8.11\times10^{10}n^3\cos 0.251n$$
$$-1.94\times10^{12}n^2\sin 0.251n + 2.45\times10^{14}\sin 0.251n - 3.08\times10^{13}n\cos 0.251n$$
$$+8.11\times10^{10}n^3 - 3.08\times10^{13}n)/n^5\cos\frac{n\pi t}{500}$$
$$+\sum_{n=1}^{\infty}(4.0\times10^{-10}(5.57\times10^{8}n^4\cos 0.251n - 8.11\times10^{10}n^3\sin 0.251n$$
$$-1.94\times10^{12}n^2\cos 0.251n + 2.45\times10^{14}\cos 0.251n + 3.08\times10^{13}n\sin 0.251n$$
$$-2.45\times10^{14} - 5.57\times10^{8}n^4 + 1.94\times10^{12}n^2)/n^5)\sin\frac{n\pi t}{500}$$

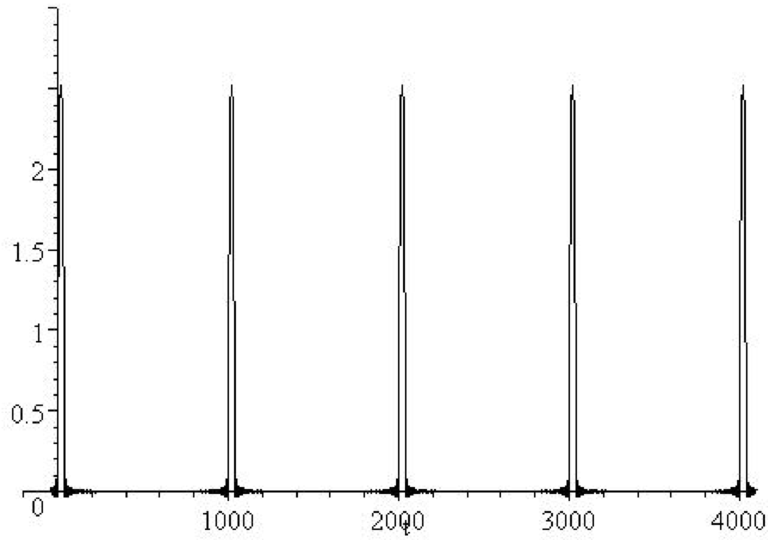
When we graph the first 5 terms of this expression, we get the following. It shows regular peaks every one second:



We take more terms to get a better graph. Here is the graph of the first 20 terms, and it's starting to look more like the required R wave:



And now the graph of the first 100 terms:



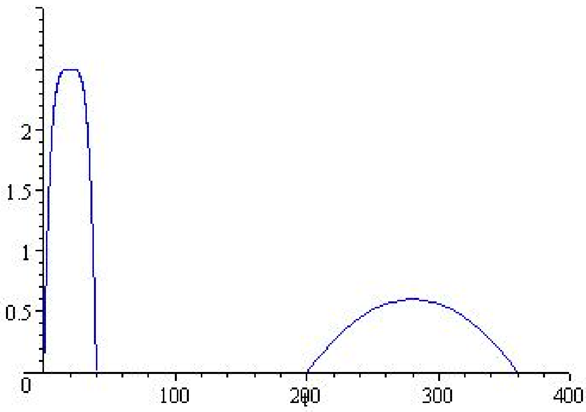
Extended Model (adding the T wave)

Our function is:

$$f(t) = \begin{cases} -0.0000156(t-20)^4 + 2.5 & \text{if} \quad 0 < t < 40 \\ -9.375 \times 10^{-5}(t-280)^2 + 0.6 & \text{if} \quad 200 < t < 360 \end{cases}$$

$$f(t) = (t + 1000)$$

The graph of the function:



Mean value:

$$a_0 = \frac{1}{L}\int_{-L}^{L} f(t)dt$$
$$= \frac{1}{500}\int_{-500}^{500} f(t)dt$$
$$= \frac{1}{500}\left(\int_0^{40}(-0.0000156(t-20)^4 + 2.5)dt + \int_{200}^{360}(-9.375\times10^{-5}(t-280)^2 + 0.6)dt\right)$$
$$= 0.288$$

Coefficient an:

$$a_n = \frac{1}{L}\int_{-L}^{L} f(t)\cos\frac{n\pi t}{L}dt$$
$$= \frac{1}{500}\left(\int_0^{40}(-0.0000156(t-20)^4 + 2.5)\cos\frac{n\pi t}{500}dt + \int_{200}^{360}(-9.375\times10^{-5}(t-280)^2 + 0.6)\cos\frac{n\pi t}{500}dt\right)$$

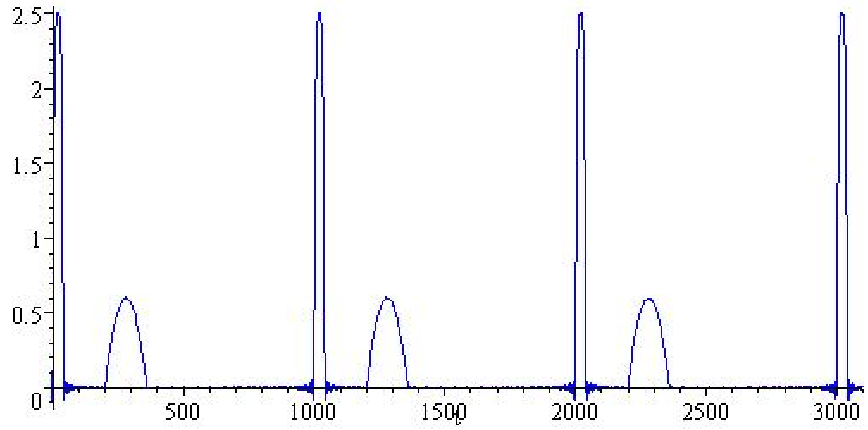
Coefficient bn:

$$b_n = \frac{1}{L}\int_{-L}^{L} f(t)\sin\frac{n\pi t}{L}dt$$
$$= \frac{1}{500}\left(\int_0^{40}(-0.0000156(t-20)^4 + 2.5)\sin\frac{n\pi t}{500}dt + \int_{200}^{360}(-9.375\times10^{-5}(t-280)^2 + 0.6)\sin\frac{n\pi t}{500}dt\right)$$

Fourier Series:

$$
\begin{aligned}
f(t) &= \frac{a_0}{2} + \sum_{n-1}^{\infty} a_n \cos\frac{n\pi t}{L} + \sum_{n-1}^{\infty} b_n \sin\frac{n\pi t}{L} \\
&= \frac{0.288}{2} \\
&\quad + \sum_{n-1}^{\infty} \frac{1}{500}\Big(\int_0^{40} (-0.0000156(t-20)^4 + 2.5)\cos\frac{n\pi t}{500}dt \\
&\quad + \int_{200}^{360} (-9.375\times 10^{-5}(t-280)^2 + 0.6)\cos\frac{n\pi t}{500}dt\Big)\cos\frac{n\pi t}{500} \\
&\quad + \sum_{n-1}^{\infty} \frac{1}{500}\Big(\int_0^{40} (-0.0000156(t-20)^4 + 2.5)\sin\frac{n\pi t}{500}dt \\
&\quad + \int_{200}^{360} (-9.375\times 10^{-5}(t-280)^2 + 0.6)\sin\frac{n\pi t}{500}dt\Big)\sin\frac{n\pi t}{500}
\end{aligned}
$$

Here is the graph:



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